Eric Rouse

Individual Assignments #58

Assignment: 4.3: 12, 26c ("5 | a+b" meas that 5 divides a+b), 43, 44; pg. 332 problem 58

# Q12

By Definition of Fibonacci:  
f0 = 0  
f1 = 1  
fn+1 = fn + fn-1

BASE: n=1;  
f12 = f1\* f01 = 1(f1 + f0 )=1\*(1+1) = 1  
1 = 1; base case checks out

INDUCTIVE:  
Assume m and m-1 holds, show that m+1 holds true also.  
Trying to prove that .  
Find the sum of m+1:  
  
Substitute m+1 into right side of equation to prove:  
  
Substitute fm+2 by definition of Fibonacci sequence.  
  
Expand:  
  
Which is the same as what we assumed the sum of m+1 would be. We have shown the base and inductive cases, thus by induction it is proven.

# Q26c

BASE: (0+0)/5 = 0; check.

INDUCTIVE: Since we add 2 to one member of the pair and 3 to the other member in all cases at each recursive step we are in essence adding five to the sum a+b. Thus if a+b starts out divisible by five (base case) then it will always be divisible by 5.

# Q43

Definitions of set T binary trees:  
BASE: single node ∊T  
RECURSIVE: If T1 and T2 are full binary trees then T=T1\*T2 has a height h(T)=1+max(h(T1),h(T2))

Prove h(T) ≥ 2h(T) + 1

BASE: h(T) = 0  
1≥2\*0+1  
1≥1 check!

INDUCTIVE:  
Assume h(T1)≥2(h(T1))+1 and h(T2)≥2(h(T2))+1 where T1 and T2 are binary trees.  
n(T) = 1 + n(T1) + n(T2) and h(T) = 1 + max(h(T1),h(T2))  
n(T) ≥ 1 + (2(h(T1))+1)+(2(h(T2))+1)  
n(T) ≥ 3 + 2(h(T1)+h(T2)) = 3 +2(max(h(T1),h(T2)))  
n(T) ≥ 3 + 2(h(T)-1) = 3+2h(T)-2  
n(T) ≥ 2h(T)+1 check!

# Q44

BASE: For any connected node with leaves there are two leaves and a single vertices.  
RECURESIVE: if T1 and T2 are full binary rees then T=T1\*T2 has n(T)=1+n(T1)+n(T2) nodes and n(T) = i(T) + l(T) and i(T) = 1+i(T1)+i(T2) and l(T) = l(T1) + l(T2)

Show that l(T) = i(T) + 1

BASE: single node, leaves = 1, vertices = 0. Check!

RECURSIVE:  
n(T) = 1 + n(T1) + n(T2)  
n(T) = 1 + i(T1)+l(T1)+i(T2)+l(T2)  
substitute l(T) = i(T) +1 to see if it equals 1 + n(T1) + n(T2)  
n(T) = 1+i(T1)+i(T1)+1+i(T2)+i(T2)+1  
simplify, recognize that n(T1) = i(T1) + l(T1):  
n(T) = 1 + n(T1) + n(T2) Check!

Thus it is shown by induction that the number of leaves equals the number of internal vertices plus 1.

# P332Q58

BASE: λ is empty which means there are zero ( and zero ). Check!

RECURSIVE: Assume x∊B equal to (,). Adding (() and ()) to the string results in an equal amount of parenthesis. Thus it is inductively proven.